

What is claimed is:

- 1 1. A method for obtaining a global optimal solution of general nonlinear programming  
2 problems, comprising the steps of:
  - 3 a) in a deterministic manner, first finding all local optimal solutions; and
  - 4 b) then finding from said local optimal solutions a global optimal solution.
- 1 2. A method for obtaining a global optimal solution of general nonlinear programming  
2 problems, comprising the steps of:
  - 3 a) in a deterministic manner, first finding all stable equilibrium points of a  
4 nonlinear dynamical system that satisfies conditions (C1) and (C2);  
5 and
  - 6 b) then finding from said points a global optimal solution.
- 1 3. A practical numerical method for reliably computing a dynamical decomposition point  
2 for large-scale systems, comprising the steps of:
  - 3 a) moving along a search path  $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, \quad t \in \mathbb{R}^+\}$  starting from  
4  $x_s$  and detecting an exit point,  $x_{ex}$ , at which said search path  $\varphi_t(x_s)$   
5 exits a stability boundary of a stable equilibrium point  $x_s$ ;
  - 6 b) using said exit point  $x_{ex}$  as an initial condition and integrating a  
7 nonlinear system (4.2) to an equilibrium point  $x_d$ ; and
  - 8 c) computing said dynamical decomposition point with respect to a local  
9 optimal solution  $x_s$  wherein said search direction  $\hat{s}$  is  $e_{x_d}$ .
- 1 4. The method of claim 3, wherein a method for computing said exit point comprises the  
2 step of:
  - 3 moving along said search path  $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, \quad t \in \mathbb{R}^+\}$  starting from  
4  $x_s$  and detecting said exit point  $x_{ex}$ , which is a first local maximum  
5 of an objective function  $C(x)$  along said search path  $\varphi_t(x_s)$ .
- 1 5. The method of claim 3, wherein a method for computing a minimum distance point  
2 (MDP) comprises the steps of:

- a) using said exit point  $x_{ex}$  as an initial condition and integrating a nonlinear system (4.2) to a first local minimum of a norm  $\|F(x)\|$ , where  $F(x)$  is a vector of a vector field (4.2), and a local minimum point is  $x_d^0$ ;
- b) using said MDP  $x_{d,j}^{t,0}$  as an initial guess and solving a set of nonlinear algebraic equations of said vector field (4.2)  $F(x) = 0$ , wherein a solution is  $x_d$ , and a dynamical decomposition point with respect to the local optimal solution  $x_s$  and said search path  $\varphi_t(x_s)$  is  $x_d$ .

6. The method of claim 3, wherein a method for computing said exit point comprises the step of computing an inner-product of said search vector and vector field at each time step, by moving along said search path  $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, \quad t \in \mathbb{R}^+\}$  starting from  $x_s$  and at each time-step, computing an inner-product of said search vector  $\hat{s}$  and vector field  $F(x)$ , such that when a sign of said inner-product changes from positive to negative, said exit point is detected.

7. The method of claim 3, wherein a method for computing said exit point comprises the step of:

- a) moving along said search vector until an inner-product changes sign between an interval  $[t_1, t_2]$ ;
- b) applying a linear interpolation to an interval  $[t_1, t_2]$ , which produces an intermediate time  $t_0$  where an interpolated inner-product is expected to be zero;
- c) computing an exact inner-product at  $t_0$ , such that if said value is smaller than a threshold value, said exit point is obtained; and
- d) if said inner-product is positive, then replacing  $t_1$  with  $t_0$ , and otherwise replacing  $t_2$  with  $t_0$  and going to step b).

8. The method of claim 3, wherein a method for computing a minimum distance point (MDP) comprises the steps of:

- a) using said exit point as an initial condition and integrating a nonlinear system for a few time-steps;

- b) checking convergence criterion, and, if a norm of said exit point obtained in step a) is smaller than a threshold value, then said point is declared as said MDP, and otherwise, going to step b);
  - c) drawing a ray connecting a current point on a trajectory and a local optimal solution, replacing said current point with a corrected exit point, which is a first local maximal point of objective function along said ray, starting from a stable equilibrium point, and assigning this point to said exit point and going to step a).
9. The method of claim 3, wherein a method for computing said dynamical decomposition point with respect to a stable equilibrium point  $x_s$  and a search vector  $\hat{s}$ , comprises the steps of:
- a) moving along said search path  $\varphi_t(x_s) = \{x_s + t \times \hat{s}, t \in \mathbb{R}^+\}$  starting from  $x_s$  and detecting a moment that an inner-product of said search vector  $\hat{s}$  and vector field  $F(x)$  changes sign, between an interval  $[t_1, t_2]$ , stopping this step if  $t_1$  is greater than a threshold value and reporting that there is no adjacent local optimal solution along this search path, and otherwise, going to step b);
  - b) applying linear interpolation to said interval  $[t_1, t_2]$ , which produces an intermediate time  $t_0$  where said interpolated inner-product is expected to be zero, computing an exact inner-product at  $t_0$ , and if said value is smaller than a threshold value, said exit point is obtained, and going to step d);
  - c) if said inner-product is positive, then replacing  $t_1$  with  $t_0$ , and otherwise replacing  $t_2$  with  $t_0$  and going to step b);
  - d) using said exit point as an initial condition and integrating a nonlinear system for a few time-steps;
  - e) checking convergence criterion, and if a norm of said point obtained in step d) is smaller than a threshold value, then said point is declared as the MDP and going to step g), and otherwise going to step e);
  - f) drawing a ray connecting a current point on said trajectory and a local optimal solution, replacing said current point with a corrected exit

point which is a first local maximal point of objective function along said ray starting from a stable equilibrium point, and assigning this point to said exit point and going to Step d); and

g) using said MDP as an initial guess and solving a set of nonlinear algebraic equations of the vector field (4.2)  $F(x) = 0$ , wherein a solution is  $t_d$ , such that said DDP with respect to a local optimal solution  $x_s$  and vector  $\hat{s}$  is  $x_d$ .

10. The method of claim 3, wherein at least one effective local search method is combined with said dynamical trajectory method of claim 3, comprising the steps of:

- a) given an initial point, integrating a nonlinear dynamical system described by (4.2) that satisfies condition (C1) from an initial point for a few time-steps to get an end point and then updating said initial point using an endpoint, before going to step b);
- b) applying an effective local optimizer starting from said end point in step a) to continue the search process, and if it converges, then stopping, or otherwise, returning to step a).

11. The method of claim 3, wherein said method is used to accomplish a result selected from the group consisting of:

- a) escaping from a local optimal solution;
- b) guaranteeing the existence of another local optimal solution;
- c) avoiding re-visit of a local optimal solution of step b);
- d) assisting in searching a local optimal solution of step b); and
- e) guaranteeing non-existence of another adjacent local optimal solution along a search path.

12. The method of claim 3, wherein a numerical method for performing a procedure, which searches from a local optimal solution to find another local optimal solution in a deterministic manner, comprises the steps of:

- a) moving along a search path starting from  $x_{opt}$  and applying said DDP search method to compute a corresponding DDP, and going to step

- 6                   b), and if a DDP can not be found, then trying another search path  
7                   and repeating this step;
- 8                   b) letting said DDP be denoted as  $x_d$ ., and if  $x_d$  has previously been found,  
9                   then going to step a), otherwise going to step c);
- 10                  c) computing a DDP-based initial point  $x_o = x_{opt} + (1 + \varepsilon)(x_d - x_{opt})$  where  
11                    $\varepsilon$  is a small number, and applying a hybrid search method starting  
12                   from  $x_o$  to find a corresponding adjacent local optimal solution.